Discrete Random Variables

COURSE TEACHING ASSISTANTS

Kamal Ezzo kamal.ezzo@must.edu.eg

Maryam Gebraeil maryam.gebraeil@must.edu.eg

Tasneem Abdelraouf tasneem.abdelraouf@must.edu.eg

Rules

1 Expected value (Mean) — the average value of X

$$E(X) = \mu = \sum x\,P(x)$$

TRICK Multiply each x by its probability, then add.

Keywords: *expected, average, mean*

2 Variance, Standard Deviation and CV — how spread out X is

$$V(X) = \sum x^2 P(x) - (\text{Mean})^2$$

$$\text{SD} = \sqrt{V(X)} = \sqrt{\sum x^2 P(x) - (\text{Mean})^2}$$

$$\text{CV} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100\,\%$$

TRICK Square x, multiply by $P(x)$, add, then take away the mean squared. Variance is never negative.

Bigger CV $\rightarrow$ more variation. Use CV to compare two variables.

Keywords: *variance, standard deviation, spread, coefficient of variation, more variable*

3 Probability distribution (PDF) — is the table a real distribution?

Every $P(x)$ is between 0 and 1: $0 \le P(x) \le 1$

All probabilities add up to 1: $\sum P(x) = 1$

TRICK Missing value (a, c)? Add the table and make it equal 1.

Keywords: *probability distribution, PDF, value of a*

4 Cumulative Distribution Function (CDF) $F(x)$

$$F(x) = P(X \leq x)$$

$$P(X > a) = 1 - F(a)$$

$$P(a < X \leq b) = F(b) - F(a)$$

TRICK Add the probabilities from the left up to x. The graph is steps that go up to 1.

Keywords: *CDF, cumulative, $F(x)$*

5 Rules of E and V a, b are numbers; X, Y independent

	Expected $E(X)$	**Variance** $V(X)$
Constant	$E(5) = 5$ *the same*	$V(5) = 0$ *zero*
Number with X	$E(5X) = 5\,E(X)$	$V(5X) = 5^2\,V(X)$
Both	$E(aX + b) = a\,E(X) + b$	$V(aX + b) = a^2\,V(X)$
Minus sign	$E(3X - 4) = 3\,E(X) - 4$	$V(3X - 4) = 3^2\,V(X)$
Two variables	$E(aX - bY) = aE(X) - bE(Y)$	$V(aX - bY) = a^2V(X) + b^2V(Y)$

TRICK Variance: square the number, drop the constant, minus becomes plus. Expected: keep everything as it is.

Same X twice? Simplify first: $V(5X - 2X) = V(3X) = 9\,V(X)$.

6 Words to signs

less than	$X < 2$	more than	$X > 2$
at least	$X \geq 2$	at most	$X \leq 2$
exactly	$X = 2$	zero to 2	$0 \leq X \leq 2$

TRICK At least = this number or more. At most = this number or less.

Q1

Let X be a random variable with $E(X) = 2.666$ and $V(X) = 1.8364$. Use this data to find:

1. $E(3X)$
2. $V(3X)$
3. $E(4X + 5)$
4. $V(4X + 5)$
5. $E(3X - 1X)$
6. $V(5X - 2X)$
7. $E(3X - 4)$
8. $V(3X - 10)$
9. $V(10X)$
10. $E(5X)$
11. The coefficient of variation

Q2

The table shows the number of breakdowns per week of a machine.

Breakdowns per week (x)	**0**	**1**	**2**	**3**
Probability $P(x)$	0.15	0.2	0.35	0.3

1. Is this table a probability distribution?
2. Find the probability of the following:
   a) Zero to 2
   b) More than 1
   c) At most 2
3. Find the mean and the standard deviation.
4. Find $V(3X - 6X)$ and $E(5 - 10X)$.
5. Using the CDF, find $P(X > 2)$, and graph the CDF.
6. Draw the histogram.

Q3

The following data represent the number of expatriate students in 40 schools in 6th of October City.

Class	0–20	20–40	40–60	60–80	80–100	100–120
Frequency	5	13	9	6	5	2

Find:

1. The expected value
2. The variance
3. The standard deviation
4. The coefficient of variation
5. The CDF at $X = 70$
6. $P(X > 70)$
7. $E(4X - 5)$ and $V(3X - 6)$

Q4

x	1	2	3	4	5	6
$P(x)$	0.1	0.02	0.3	a	0.4	0.08

1. The value of a that makes the table a probability distribution (PDF) is
2. $P(X \leq 1)$
3. $P(X \geq 0.02)$
4. $F(X = 4)$
5. $E(X)$

Q5

Let X and Y be two independent random variables such that $E(X) = 2$, $V(X) = 3$, $E(Y) = 7$ and $V(Y) = 1$.

1. The coefficient of variation of the random variable X is
2. The coefficient of variation of the random variable Y is
3. Which random variable is more variable than the other?
4. $E(3X + 4)$
5. $V(3X + 2Y - 5)$

Q6

x	0	1	2	3	4	5
$P(x)$	0.2	0.15	$2c$	0.05	0.15	c

1. Find the value of c.
2. Find the following probabilities:
   a) $P(X > 1)$
   b) $P(X \leq 3)$
   c) Exactly 2
   d) Zero to 3
   e) More than 2
   f) At most 4
3. Find the cumulative distribution function (CDF), and graph it.
4. Using the CDF, find $P(1.1 < X < 3.5)$ and $P(X > 3)$.
5. Find the expected value, the variance and the standard deviation.
6. Find $V(3X - 3)$ and $E(5X - 10)$.
7. Draw the histogram.

Q7

A coin is tossed 3 times. Let X be the number of heads. Find:

1. The probability distribution function
2. The cumulative distribution function (CDF), and graph it
3. The expected value, the variance and the standard deviation

Q8

Let X be a random variable with values $\{0, 1, 2, 3\}$ and probability distribution function

$$P(x) = \frac{x^2 + 1}{18}, \qquad x = 0, 1, 2, 3$$

Find the following:

1. $P(X \leq 1)$
2. $P(0 < X < 3)$
3. The arithmetic mean and the variance

Q9

If $P(x) = \dfrac{1}{x+3}$, $x = 0, 1, 2, 3, 4$, determine if it is a PDF or not.