Special Cases and Tables

COURSE TEACHING ASSISTANTS

Kamal Ezzo kamal.ezzo@must.edu.eg

Maryam Gebraeil maryam.gebraeil@must.edu.eg

Tasneem Abdelraouf tasneem.abdelraouf@must.edu.eg

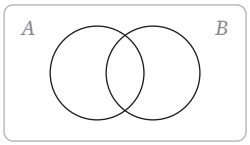
Rules

1 Independent — one does not affect the other

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

$$P(A/B) = P(A) \qquad P(B/A) = P(B)$$



TRICK Independent → multiply.

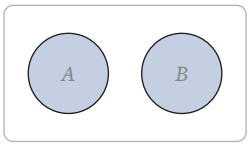
Keywords: *independent, does not affect*

2 Mutually exclusive — cannot happen together

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A - B) = P(A) \qquad P(B - A) = P(B)$$



TRICK Mutually exclusive → ∩ is zero.

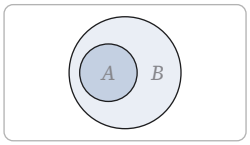
Keywords: *mutually exclusive, disjoint, cannot happen together*

3 Subset ($A \subset B$) — A is inside B

$$P(A \cap B) = P(A)$$

$$P(A \cup B) = P(B)$$

$$P(A - B) = 0$$

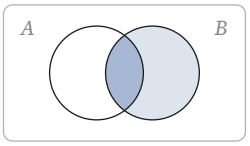


TRICK Subset → ∩ is the small one, ∪ is the big one.

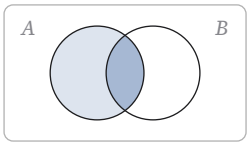
Keywords: *subset, inside*

4 Given (Conditional) A/B or $A \mid B$

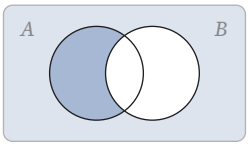
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



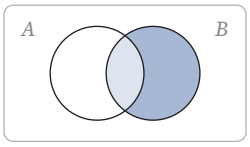
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$



$$P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})}$$



$$P(\overline{A}/B) = \frac{P(\overline{A} \cap B)}{P(B)}$$



From the same rule

$$P(A \cap B) = P(A/B) \times P(B) \qquad P(A \cap B) = P(B/A) \times P(A)$$

TRICK The event after the (/) is what we know happened. It goes in the bottom. light = bottom, dark = top

In a table: divide by the total of the given row or column, not the grand total.

Keywords: *given, if, knowing that, has occurred*

Q1

Let A, B and C be independent events of a sample space, with $P(A) = 0.2$, $P(B) = 0.46$ and $P(C) = 0.15$. Find:

1. $P(A \text{ and } B)$
2. $P(B \text{ and } C)$
3. $P(A/B)$
4. $P(C/B)$
5. $P(A \text{ and } B \text{ and } C)$
6. $P(A \cap \overline{B})$

Q2

The sample space has 10 elements, $A = \{a, b, c, d, e\}$ and $B = \{a, c, f, b, d, e, g, h\}$. Find:

1. $P(\overline{A} \cap \overline{B})$
2. $P(A/B)$
3. $P(B/A)$
4. $P(A \cap \overline{B})$
5. $P(\overline{A} \cap B)$

Q3

Event A is the letters of the word **Word**, and event B is the letters of the words **Word Press**. The sample space has 10 elements. Find:

1. $P(\overline{A} \cap \overline{B})$
2. $P(A/B)$
3. $P(B/A)$
4. $P(A \cap \overline{B})$
5. $P(\overline{A} \cap B)$

Q4

A and B are two events of a sample space such that $A \subset B$, $P(A) = \frac{1}{2}$ and $P(B - A) = 0.2$. Find:

1. The probability that B does not occur
2. The probability of A only

Q5

$X \subset Y$. Suppose $P(X) = 0.04$ and $P(Y) = 0.5$. Find $P(Y/X)$.

Q6

$X \subset Y$. Suppose $P(X) = 0.04$ and $P(Y) = 0.5$. Find $P(X/Y)$.

Q7

Is $P(A/B) = P(A)$ if $A \subset B$?

Q8

If $P(A) = 0.3$, $P(A/B) = 0.7$ and A is a subset of B, find $P(B)$.

Q9

If $P(A) = 0.3$, $P(B/A) = 0.7$ and A and B are independent events, find $P(B)$.

Q10

If $P(A) = 0.3$, $P(B/A) = 0.7$ and A and B are independent events, find $P(B \text{ or } A)$.

Q11

Suppose $P(A) = 0.4$ and $P(B/A) = 0.3$. Find the joint probability of A and B.

Q12

X and Y are independent events. Suppose $P(X) = 0.05$ and $P(Y) = 0.02$. Find the probability that Y occurs if X occurred.

Q13

Is $P(B/A) = P(A)$ if A and B are independent events?

Q14

Is $P(B/A) = P(B)$ if A and B are independent events?

Q15

A and B are mutually exclusive. Suppose $P(A) = 0.30$ and $P(B) = 0.20$.

1. What is the probability of A only?
2. What is the probability of A or B occurring?
3. What is the probability that neither A nor B will happen?
4. What is the probability of either A or B occurring?
5. What is the probability of B only?

Q16

X and Y are mutually exclusive. Suppose $P(X) = 0.03$ and $P(Y) = 0.02$. Find the probability that X occurs if Y occurred.

Q17

A single die is rolled. Are the outcomes equally likely and mutually exclusive?

Q18

A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Q19

A survey of students in the Information Systems, Business and Accounting departments gave the following results:

	Male	Female
Business	25	19
Accounting	14	12
Information Systems	11	9

A student is selected at random. Find the probability that the student is:

1. From the Business department
2. From the Accounting department and male
3. From Information Systems or female
4. From Business and Accounting
5. From Business or Accounting
6. Are Business and Male independent events?

Q20

	College Graduate	Not College Graduate	Total
Smoker	35	80	**115**
Non-Smoker	130	175	**305**
Total	**165**	**255**	**420**

An employee is selected at random. Find the probability that he is:

1. A college graduate
2. A non-smoker
3. A smoker, given he is not a college graduate
4. A non-smoker and a college graduate
5. A college graduate and a smoker
6. A non-smoker or not a college graduate
7. Are Non-Smoker and College Graduate mutually exclusive? What about Smoker and Non-Smoker? Why?
8. Are Smoker and Not College Graduate independent? Why?

Q21

Students were asked about an issue (Yes = favors, No = against):

	Yes	No	No Opinion
Freshmen	20	13	120
Sophomores	18	22	10

A student is selected at random. Find the probability that the student:

1. Has no opinion
2. Is a freshman or is against the issue
3. Is a sophomore and favors the issue
4. Is a freshman and a sophomore
5. Is a freshman or a sophomore
6. Is a freshman, given he has no opinion
7. Is against the issue, given he is a sophomore

Q22

	Under 35	35 to 54	Over 54	Total
Male	27	87	26	**140**
Female	14	25	3	**42**
Total	**41**	**112**	**29**	**182**

A person is selected at random. Find the probability that the person is:

1. Female and over 54
2. Male or between 35 and 54
3. Male, given under 35
4. Are Male and Female mutually exclusive events?