Special Cases and Tables

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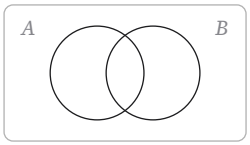
Rules

1 Independent — one does not affect the other

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

$$P(A/B) = P(A) \qquad P(B/A) = P(B)$$



TRICK Independent → multiply.

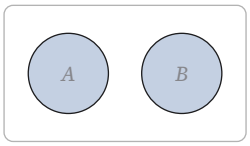
Keywords: *independent, does not affect*

2 Mutually exclusive — cannot happen together

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A - B) = P(A) \qquad P(B - A) = P(B)$$



TRICK Mutually exclusive → ∩ is zero.

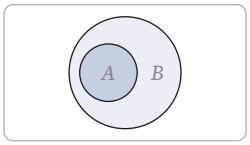
Keywords: *mutually exclusive, disjoint, cannot happen together*

3 Subset ($A \subset B$) — A is inside B

$$P(A \cap B) = P(A)$$

$$P(A \cup B) = P(B)$$

$$P(A - B) = 0$$

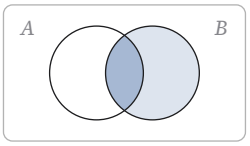


TRICK Subset → ∩ is the small one, ∪ is the big one.

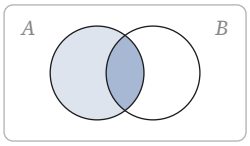
Keywords: *subset, inside*

4 Given (Conditional) A/B or $A \mid B$

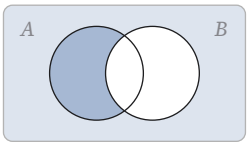
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$



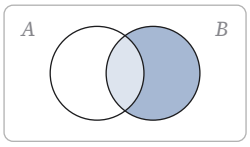
$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$



$$P(A/\overline{B}) = \frac{P(A \cap \overline{B})}{P(\overline{B})}$$



$$P(\overline{A}/B) = \frac{P(\overline{A} \cap B)}{P(B)}$$



From the same rule

$$P(A \cap B) = P(A/B) \times P(B) \qquad P(A \cap B) = P(B/A) \times P(A)$$

TRICK The event after the (/) is what we know happened. It goes in the bottom. light = bottom, dark = top

Keywords: *given, if, knowing that, has occurred*

5 Tables

	Yes	No	Total
Male	30	20	**50**
Female	10	40	**50**
Total	**40**	**60**	**100**

One $P(\text{Male}) = \dfrac{\text{row total}}{\text{grand total}} = \dfrac{50}{100}$

And $P(\text{Male} \cap \text{Yes}) = \dfrac{\text{the cell}}{\text{grand total}} = \dfrac{30}{100}$

Or $P(\text{Male} \cup \text{Yes}) = \dfrac{\text{row} + \text{column} - \text{cell}}{\text{grand total}} = \dfrac{50 + 40 - 30}{100}$

Given $P(\text{Yes/Male}) = \dfrac{\text{the cell}}{\text{total of the given}} = \dfrac{30}{50}$

TRICK Given → divide by the total of the given row or column, not the grand total.

Keywords: *two-way table, contingency table*

Q1

Let A, B and C be independent events of a sample space, with $P(A) = 0.2$, $P(B) = 0.46$ and $P(C) = 0.15$. Find:

1. $P(A \text{ and } B)$
2. $P(B \text{ and } C)$
3. $P(A/B)$
4. $P(C/B)$
5. $P(A \text{ and } B \text{ and } C)$
6. $P(A \cap \overline{B})$

Solution

Given: Independent: $P(A) = 0.2$, $P(B) = 0.46$, $P(C) = 0.15$

1) $P(A \text{ and } B)$

$$P(A \cap B) = P(A) \times P(B)$$

$$= 0.2 \times 0.46 = 0.092$$

2) $P(B \text{ and } C)$

$$P(B \cap C) = P(B) \times P(C)$$

$$= 0.46 \times 0.15 = 0.069$$

3) $P(A/B)$

$$P(A/B) = P(A) = 0.2$$ *independent*

4) $P(C/B)$

$$P(C/B) = P(C) = 0.15$$ *independent*

5) $P(A \text{ and } B \text{ and } C)$

$$P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$$

$$= 0.2 \times 0.46 \times 0.15 = 0.0138$$

6) $P(A \cap \overline{B})$

$$P(A \cap \overline{B}) = P(A - B) = P(A) - P(A \cap B)$$

$$= 0.2 - 0.092 = 0.108$$

$P(A \cap B) = 0.092$ *from part 1*

Q2

The sample space has 10 elements, $A = \{a, b, c, d, e\}$ and $B = \{a, c, f, b, d, e, g, h\}$. Find:

1. $P(\overline{A} \cap \overline{B})$
2. $P(A/B)$
3. $P(B/A)$
4. $P(A \cap \overline{B})$
5. $P(\overline{A} \cap B)$

Solution

Given: $n(S) = 10$, $n(A) = 5$, $n(B) = 8$

$$A \cap B = \{a, b, c, d, e\}, \quad n(A \cap B) = 5$$

$$A \cup B = \{a, b, c, d, e, f, g, h\}, \quad n(A \cup B) = 8$$

Every element of A is in B, so $A \subset B$.

$$P(A) = 0.5, \quad P(B) = 0.8, \quad P(A \cap B) = 0.5, \quad P(A \cup B) = 0.8$$

1) $P(\overline{A} \cap \overline{B})$

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - 0.8 = 0.2$$

2) $P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.5}{0.8} = 0.625$$

3) $P(B/A)$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.5}{0.5} = 1$$

4) $P(A \cap \overline{B})$

$$P(A \cap \overline{B}) = P(A - B) = P(A) - P(A \cap B)$$

$$= 0.5 - 0.5 = 0$$ *$A \subset B$, so nothing of A is outside B*

5) $P(\overline{A} \cap B)$

$$P(\overline{A} \cap B) = P(B - A) = P(B) - P(A \cap B)$$

$$= 0.8 - 0.5 = 0.3$$

Q3

Event A is the letters of the word **Word**, and event B is the letters of the words **Word Press**. The sample space has 10 elements. Find:

1. $P(\overline{A} \cap \overline{B})$
2. $P(A/B)$
3. $P(B/A)$
4. $P(A \cap \overline{B})$
5. $P(\overline{A} \cap B)$

Solution

Given: $n(S) = 10$

$A = \{w, o, r, d\}, \quad n(A) = 4$

$B = \{w, o, r, d, p, e, s\}, \quad n(B) = 7$

A repeated letter is counted once. Every letter of A is in B, so $A \subset B$.

$P(A) = 0.4, \quad P(B) = 0.7, \quad P(A \cap B) = 0.4, \quad P(A \cup B) = 0.7$

1) $P(\overline{A} \cap \overline{B})$

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - 0.7 = 0.3$$

2) $P(A/B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.7} \approx 0.57$$

3) $P(B/A)$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{0.4}{0.4} = 1$$

4) $P(A \cap \overline{B})$

$$P(A \cap \overline{B}) = P(A - B) = P(A) - P(A \cap B)$$

$$= 0.4 - 0.4 = 0$$

5) $P(\overline{A} \cap B)$

$$P(\overline{A} \cap B) = P(B - A) = P(B) - P(A \cap B)$$

$$= 0.7 - 0.4 = 0.3$$

Q4

A and B are two events of a sample space such that $A \subset B$, $P(A) = \frac{1}{2}$ and $P(B - A) = 0.2$. Find:

1. The probability that B does not occur
2. The probability of A only

Solution

Given: $A \subset B$, $P(A) = 0.5$, $P(B - A) = 0.2$

First, find $P(B)$

$$P(B - A) = P(B) - P(A \cap B) \qquad A \subset B,\ so\ P(A \cap B) = P(A)$$

$$0.2 = P(B) - 0.5$$

$$P(B) = 0.2 + 0.5 = 0.7$$

1) *B does not occur*

$$P(\overline{B}) = 1 - P(B)$$

$$= 1 - 0.7 = 0.3$$

2) *A only*

$$P(A - B) = P(A) - P(A \cap B)$$

$$= 0.5 - 0.5 = 0 \qquad A\ is\ inside\ B$$

Q5

$X \subset Y$. Suppose $P(X) = 0.04$ and $P(Y) = 0.5$. Find $P(Y/X)$.

Solution

Given: $X \subset Y$, so $P(X \cap Y) = P(X) = 0.04$

$P(Y/X)$

$$P(Y/X) = \frac{P(X \cap Y)}{P(X)} = \frac{0.04}{0.04} = 1$$

If X happened, Y surely happened: X is inside Y.

Q6

$X \subset Y$. Suppose $P(X) = 0.04$ and $P(Y) = 0.5$. Find $P(X/Y)$.

Solution

Given: $X \subset Y$, so $P(X \cap Y) = P(X) = 0.04$

$P(X/Y)$

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{0.04}{0.5} = 0.08$$

Q7

Is $P(A/B) = P(A)$ if $A \subset B$?

Solution

Use the given rule

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} \quad A \subset B, \text{ so } P(A \cap B) = P(A)$$

No $P(A/B) = \dfrac{P(A)}{P(B)}$, not $P(A)$. *$P(A/B) = P(A)$ is the rule for independent events.*

Q8

If $P(A) = 0.3$, $P(A/B) = 0.7$ and A is a subset of B, find $P(B)$.

Solution

Given: $A \subset B$, so $P(A \cap B) = P(A) = 0.3$

$P(B)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$0.7 = \frac{0.3}{P(B)}$$

$$P(B) = \frac{0.3}{0.7} \approx 0.43$$

Q9

If $P(A) = 0.3$, $P(B/A) = 0.7$ and A and B are independent events, find $P(B)$.

Solution

Given: Independent: $P(A) = 0.3$, $P(B/A) = 0.7$

$P(B)$

$P(B) = P(B/A) = 0.7$ *independent: $P(B/A) = P(B)$*

Q10

If $P(A) = 0.3$, $P(B/A) = 0.7$ and A and B are independent events, find $P(B \text{ or } A)$.

Solution

Given: Independent: $P(A) = 0.3$, $P(B) = P(B/A) = 0.7$ *as in Q9*

First, find $P(A \cap B)$

$$P(A \cap B) = P(A) \times P(B)$$

$$= 0.3 \times 0.7 = 0.21$$

Then, $P(B \text{ or } A)$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.7 - 0.21 = 0.79$$ $P(A \cap B) = 0.21$ *from the first step*

Q11

Suppose $P(A) = 0.4$ and $P(B/A) = 0.3$. Find the joint probability of A and B.

Solution

Given: $P(A) = 0.4$, $P(B/A) = 0.3$ *joint = and*

$P(A \cap B)$

$$P(A \cap B) = P(B/A) \times P(A)$$

$$= 0.3 \times 0.4 = 0.12$$

Q12

X and Y are independent events. Suppose $P(X) = 0.05$ and $P(Y) = 0.02$. Find the probability that Y occurs if X occurred.

Solution

Given: Independent: $P(X) = 0.05$, $P(Y) = 0.02$ *if X occurred = given X*

$P(Y/X)$

$P(Y/X) = P(Y) =$ 0.02 *independent*

Q13

Is $P(B/A) = P(A)$ if A and B are independent events?

Solution

No For independent events, $P(B/A) = P(B)$, not $P(A)$.

Q14

Is $P(B/A) = P(B)$ if A and B are independent events?

Solution

Yes A does not affect B, so knowing A happened does not change $P(B)$.

Q15

A and B are mutually exclusive. Suppose $P(A) = 0.30$ and $P(B) = 0.20$.

1. What is the probability of A only?
2. What is the probability of A or B occurring?
3. What is the probability that neither A nor B will happen?
4. What is the probability of either A or B occurring?
5. What is the probability of B only?

Solution

Given: Mutually exclusive: $P(A) = 0.3$, $P(B) = 0.2$, $P(A \cap B) = 0$

1) *A only*

$$P(A - B) = P(A) = 0.3$$ *mutually exclusive*

2) *A or B*

$$P(A \cup B) = P(A) + P(B)$$

$$= 0.3 + 0.2 = 0.5$$

3) *Neither A nor B*

$$P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$$

$$= 1 - 0.5 = 0.5$$ $P(A \cup B) = 0.5$ *from part 2*

4) *Either A or B*

$$P(A \cup B) = 0.5$$ *same as part 2*

5) *B only*

$$P(B - A) = P(B) = 0.2$$ *mutually exclusive*

Q16

X and Y are mutually exclusive. Suppose $P(X) = 0.03$ and $P(Y) = 0.02$. Find the probability that X occurs if Y occurred.

Solution

Given: Mutually exclusive, so $P(X \cap Y) = 0$

$P(X/Y)$

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{0}{0.02} = 0$$

If Y happened, X cannot happen.

Q17

A single die is rolled. Are the outcomes equally likely and mutually exclusive?

Solution

Given: $S = \{1, 2, 3, 4, 5, 6\}$

Equally likely?

Yes every face has the same probability, $\frac{1}{6}$.

Mutually exclusive?

Yes only one face shows at a time, so two faces cannot happen together.

Q18

A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

Solution

Given: $P(F) = 0.98$ (fire engine), $P(M) = 0.92$ (ambulance), independent

Both available

$$P(F \cap M) = P(F) \times P(M)$$

$$= 0.98 \times 0.92 = 0.9016$$

Q19

A survey of students in the Information Systems, Business and Accounting departments gave the following results:

	Male	Female
Business	25	19
Accounting	14	12
Information Systems	11	9

A student is selected at random. Find the probability that the student is:

1. From the Business department
2. From the Accounting department and male
3. From Information Systems or female
4. From Business and Accounting
5. From Business or Accounting
6. Are Business and Male independent events?

Solution

First, add the totals

	Male	Female	Total
Business	25	19	**44**
Accounting	14	12	**26**
Information Systems	11	9	**20**
Total	**50**	**40**	**90**

1) *Business*

$$P(\text{B}) = \frac{44}{90} \approx 0.49$$

2) *Accounting and male*

$$P(\mathrm{A} \cap \mathrm{M}) = \frac{14}{90} \approx 0.16$$

3) *Information Systems or female*

$$P(\mathrm{IS} \cup \mathrm{F}) = \frac{20 + 40 - 9}{90} = \frac{51}{90} \approx 0.57$$

4) *Business and Accounting*

$P(\mathrm{B} \cap \mathrm{A}) = 0$ *one student is in one department: mutually exclusive*

5) *Business or Accounting*

$$P(\mathrm{B} \cup \mathrm{A}) = \frac{44 + 26}{90} = \frac{70}{90} \approx 0.78$$ *mutually exclusive: just add*

6) *Business and Male independent?*

$$P(\mathrm{B} \cap \mathrm{M}) = \frac{25}{90} \approx 0.28$$

$$P(\mathrm{B}) \times P(\mathrm{M}) = \frac{44}{90} \times \frac{50}{90} \approx 0.27$$

$0.28 \neq 0.27$ Not independent

Q20

	College Graduate	Not College Graduate	Total
Smoker	35	80	**115**
Non-Smoker	130	175	**305**
Total	**165**	**255**	**420**

An employee is selected at random. Find the probability that he is:

1. A college graduate
2. A non-smoker
3. A smoker, given he is not a college graduate
4. A non-smoker and a college graduate
5. A college graduate and a smoker
6. A non-smoker or not a college graduate
7. Are Non-Smoker and College Graduate mutually exclusive? What about Smoker and Non-Smoker? Why?
8. Are Smoker and Not College Graduate independent? Why?

Solution

Given: Grand total = 420

CG = college graduate, NCG = not college graduate, S = smoker, NS = non-smoker

1) *College graduate*

$$P(\text{CG}) = \frac{165}{420} \approx 0.39$$

2) *Non-smoker*

$$P(\text{NS}) = \frac{305}{420} \approx 0.73$$

3) *Smoker, given not a college graduate*

$$P(\text{S/NCG}) = \frac{80}{255} \approx 0.31$$ *divide by the NCG total*

4) *Non-smoker and college graduate*

$$P(\text{NS} \cap \text{CG}) = \frac{130}{420} \approx 0.31$$

5) *College graduate and smoker*

$$P(\text{CG} \cap \text{S}) = \frac{35}{420} \approx 0.08$$

6) *Non-smoker or not a college graduate*

$$P(\text{NS} \cup \text{NCG}) = \frac{305 + 255 - 175}{420} = \frac{385}{420} \approx 0.92$$

7) *Mutually exclusive?*

Non-Smoker and College Graduate: No they share 130 employees.

Smoker and Non-Smoker: Yes an employee cannot be both, so $P(\text{S} \cap \text{NS}) = 0$.

8) *Smoker and Not College Graduate independent?*

$$P(\text{S} \cap \text{NCG}) = \frac{80}{420} \approx 0.19$$

$$P(\text{S}) \times P(\text{NCG}) = \frac{115}{420} \times \frac{255}{420} \approx 0.17$$

$0.19 \neq 0.17$ Not independent

Q21

Students were asked about an issue (Yes = favors, No = against):

	Yes	No	No Opinion
Freshmen	20	13	120
Sophomores	18	22	10

A student is selected at random. Find the probability that the student:

1. Has no opinion
2. Is a freshman or is against the issue
3. Is a sophomore and favors the issue
4. Is a freshman and a sophomore
5. Is a freshman or a sophomore
6. Is a freshman, given he has no opinion
7. Is against the issue, given he is a sophomore

Solution

First, add the totals

	Yes	No	No Opinion	Total
Freshmen	20	13	120	**153**
Sophomores	18	22	10	**50**
Total	**38**	**35**	**130**	**203**

1) *No opinion*

$$P(\text{NO}) = \frac{130}{203} \approx 0.64$$

2) *Freshman or against*

$$P(\text{F} \cup \text{No}) = \frac{153 + 35 - 13}{203} = \frac{175}{203} \approx 0.86$$

3) *Sophomore and favors*

$$P(\text{S} \cap \text{Yes}) = \frac{18}{203} \approx 0.09$$

4) *Freshman and sophomore*

$$P(\text{F} \cap \text{S}) = 0$$ *a student cannot be both: mutually exclusive*

5) *Freshman or sophomore*

$$P(\text{F} \cup \text{S}) = \frac{153 + 50}{203} = \frac{203}{203} = 1$$

6) *Freshman, given no opinion*

$$P(\text{F/NO}) = \frac{120}{130} \approx 0.92$$ *divide by the No Opinion total*

7) *Against, given sophomore*

$$P(\text{No/S}) = \frac{22}{50} = 0.44$$ *divide by the sophomore total*

Q22

	Under 35	35 to 54	Over 54	Total
Male	27	87	26	**140**
Female	14	25	3	**42**
Total	**41**	**112**	**29**	**182**

A person is selected at random. Find the probability that the person is:

1. Female and over 54
2. Male or between 35 and 54
3. Male, given under 35
4. Are Male and Female mutually exclusive events?

Solution

Given: Grand total = 182

1) *Female and over 54*

$$P(\text{F} \cap \text{Over 54}) = \frac{3}{182} \approx 0.02$$

2) *Male or 35 to 54*

$$P(\text{M} \cup 35 \text{ to } 54) = \frac{140 + 112 - 87}{182} = \frac{165}{182} \approx 0.91$$

3) *Male, given under 35*

$$P(\text{M/Under 35}) = \frac{27}{41} \approx 0.66$$

divide by the Under 35 total

4) *Male and Female mutually exclusive?*

Yes — a person cannot be both, so $P(\text{M} \cap \text{F}) = 0$.