Annuities

COURSE TEACHING ASSISTANTS

Kamal Ezzo kamal.ezzo@must.edu.eg
Maryam Gebraeil maryam.gebraeil@must.edu.eg
Tasneem Abdelraouf tasneem.abdelraouf@must.edu.eg
Sarah Hamzah sarah.hamza@must.edu.eg

Rules

Given		Keywords
A	the annuity: the same amount paid each time	*deposits, pays, invests*
n	number of payments = term ÷ period	*every 2 months for 1 year:* $n = 12 \div 2$
r	annual simple interest rate	*% annually*
F	time from the **first** payment to the end	*months, or days*
L	time from the **last** payment to the end	

Step	Rule	Keywords
Total	$\sum A = (A \times n) + A \times r \times \frac{n}{2} \times \frac{F+L}{12}$	*accumulated amount, sum, total*
	use 360 instead of 12 when F and L are in days	
Interest	$\sum A - (A \times n)$	*interest*
Balance	$\sum A_{\text{deposits}} - \sum A_{\text{withdrawals}}$	*withdraws, balance, amount due*
Due	payment at the **beginning**: F = term, L = one period	*beginning, rent*
Ordinary	payment at the **end**: F = term − one period, $L = 0$	*end, salary*
	no type in the question means ordinary	

Note. To find A or r: put the numbers in the total rule and solve for the unknown. Round answers to 2 decimal places.

Q1

A person deposits \$500 at the end of every 3 months for 2 years at simple interest 10% annually. Find the accumulated amount at the end of the term.

Solution

Given: $A = 500,\quad n = \dfrac{24}{3} = 8,\quad r = 10\%$ *ordinary (end)*: $F = 24 - 3 = 21,\quad L = 0$

$$\sum A = (500 \times 8) + 500 \times 0.10 \times \frac{8}{2} \times \frac{21 + 0}{12}$$

$$= 4000 + 350 \;=\; \boxed{4{,}350}$$

Q2

\$800 is deposited at the beginning of every month for 10 months at 12% simple interest. Find the total accumulated.

Solution

Given: $A = 800,\quad n = 10,\quad r = 12\%$ *due (beginning)*: $F = 10,\quad L = 1$

$$\sum A = (800 \times 10) + 800 \times 0.12 \times \frac{10}{2} \times \frac{10 + 1}{12}$$

$$= 8000 + 440 \;=\; \boxed{8{,}440}$$

Q3

A person deposits a fixed sum at the end of every 6 months for 3 years at 8% simple interest. At the end of the term the accumulated amount is \$6,480. Find each payment.

Solution

Given: $\sum A = 6480,\ n = \frac{36}{6} = 6,\ r = 8\%$ *ordinary*: $F = 36 - 6 = 30,\ L = 0,\ A = ??$

$$6480 = (A \times 6) + A \times 0.08 \times \frac{6}{2} \times \frac{30 + 0}{12}$$

$$6480 = 6A + 0.6A = 6.6A$$

$$A = \frac{6480}{6.6} = 981.82$$

Q4

\$250 is deposited at the beginning of every 2 months for 1 year. The accumulated sum is \$1,605. Find the interest rate.

Solution

Given: $A = 250,\ n = \frac{12}{2} = 6,\ \sum A = 1605$ *due*: $F = 12,\ L = 2,\ r = ??$

$$1605 = (250 \times 6) + 250 \times r \times \frac{6}{2} \times \frac{12 + 2}{12}$$

$$1605 = 1500 + 875\,r$$

$$r = \frac{1605 - 1500}{875} = 0.12 = 12\%$$

Q5

$150 is deposited every 15 days for 90 days at 12% simple interest. Find the accumulated amount.

Solution

Given: $A = 150$, $n = \dfrac{90}{15} = 6$, $r = 12\%$ *ordinary, days*: $F = 90 - 15 = 75$, $L = 0$

$$\sum A = (150 \times 6) + 150 \times 0.12 \times \frac{6}{2} \times \frac{75 + 0}{360}$$

$$= 900 + 11.25 = \boxed{911.25}$$

Q6

A company deposits $5,000 at the beginning and middle of each month starting in January (6 months). It withdraws $2,000 on the 10th and 20th of each month starting in March (4 months). Interest is 6%. Find the balance at the end of June.

Solution

Deposits

Given: $A_1 = 5000$, $n = 2 \times 6 = 12$, $r = 6\%$, $F = 6$ *(1 Jan)*, $L = 0.5$ *(15 Jun)*

$$\sum A_1 = (5000 \times 12) + 5000 \times 0.06 \times \frac{12}{2} \times \frac{6 + 0.5}{12} = 60000 + 975 = 60{,}975$$

Withdrawals

Given: $A_2 = 2000$, $n = 2 \times 4 = 8$, $F = 3\frac{2}{3}$ *(10 Mar)*, $L = \frac{1}{3}$ *(20 Jun)*

$$\sum A_2 = (2000 \times 8) + 2000 \times 0.06 \times \frac{8}{2} \times \frac{3\frac{2}{3} + \frac{1}{3}}{12} = 16000 + 160 = 16{,}160$$

$$\text{Balance} = 60975 - 16160 = \boxed{44{,}815}$$

Q7

$300 is deposited at the beginning of every 2 months for 1 year. The accumulated sum was $1,931.25. Find the interest rate.

Solution

Given: $A = 300,\quad n = 6,\quad \sum A = 1931.25$ *due*: $F = 12,\quad L = 2,\quad r = ??$

$$1931.25 = (300 \times 6) + 300 \times r \times \frac{6}{2} \times \frac{12+2}{12}$$

$$1931.25 = 1800 + 1050\,r$$

$$r = \frac{131.25}{1050} = 0.125 \;=\; 12.5\%$$

Q8

A sum is deposited every 15 days for 120 days at 9% simple interest. At the end of 120 days the accumulated amount is $4,090. How much is deposited each time?

Solution

Given: $\sum A = 4090,\quad n = \frac{120}{15} = 8,\quad r = 9\%$ *ordinary, days*: $F = 105,\quad L = 0,\quad A = ??$

$$4090 = (A \times 8) + A \times 0.09 \times \frac{8}{2} \times \frac{105+0}{360}$$

$$4090 = 8A + 0.105A = 8.105A$$

$$A = \frac{4090}{8.105} \;=\; 504.63$$

Q9

A firm deposits 5,000 L.E. at the beginning of each quarter starting 1 January 2020. It withdraws 2,000 L.E. at the end of each quarter starting 31 March 2020. Find the balance at the end of December 2020 at 6% simple interest.

Solution

Deposits

Given: $A_1 = 5000,\quad n = 4,\quad r = 6\%$ *due*: $F = 12$ *(1 Jan)*, $L = 3$ *(1 Oct)*

$$\sum A_1 = (5000 \times 4) + 5000 \times 0.06 \times \frac{4}{2} \times \frac{12+3}{12} = 20000 + 750 = 20{,}750$$

Withdrawals

Given: $A_2 = 2000,\quad n = 4$ *ordinary*: $F = 9$ *(31 Mar)*, $L = 0$ *(31 Dec)*

$$\sum A_2 = (2000 \times 4) + 2000 \times 0.06 \times \frac{4}{2} \times \frac{9+0}{12} = 8000 + 180 = 8{,}180$$

$$\text{Balance} = 20750 - 8180 = \boxed{12{,}570 \text{ L.E.}}$$

Q10

A semi-annual annuity of 1,200 L.E. is paid for 3 years at 8% simple interest. Find the accumulated amount at the end of 3.5 years if the annuity is:

a) ordinary

b) due

Solution

Given: $A = 1200,\ \ n = \frac{36}{6} = 6,\ \ r = 8\%,\ \ \text{the end is at month } 42$

a) *Ordinary: first payment at month 6, last at month 36* $\Rightarrow F = 36,\ \ L = 6$

$$\sum A = (1200 \times 6) + 1200 \times 0.08 \times \frac{6}{2} \times \frac{36+6}{12} = 7200 + 1008 \ = \ 8{,}208$$

b) *Due: first payment at month 0, last at month 30* $\Rightarrow F = 42,\ \ L = 12$

$$\sum A = (1200 \times 6) + 1200 \times 0.08 \times \frac{6}{2} \times \frac{42+12}{12} = 7200 + 1296 \ = \ 8{,}496$$

Q11 Extra

A person deposited 300 L.E. at the beginning of each month for the first 6 months of the year, then 600 L.E. at the end of every 2 months for the rest of the year, at a simple interest rate of 10% annually. Find the amount at the end of the year.

Solution

First group: January to June

Given: $A_1 = 300,\quad n = 6,\quad r = 10\%$ *due*: $F = 12$ *(1 Jan)*, $L = 7$ *(1 Jun)*

$$\sum A_1 = (300 \times 6) + 300 \times 0.10 \times \frac{6}{2} \times \frac{12+7}{12} = 1800 + 142.5 = 1{,}942.5$$

Second group: July to December

Given: $A_2 = 600,\quad n = \frac{6}{2} = 3$ *ordinary*: $F = 4$ *(end of Aug)*, $L = 0$ *(end of Dec)*

$$\sum A_2 = (600 \times 3) + 600 \times 0.10 \times \frac{3}{2} \times \frac{4+0}{12} = 1800 + 30 = 1{,}830$$

$$\text{Total} = 1942.5 + 1830 \;=\; \boxed{3{,}772.5 \text{ L.E.}}$$

Q12 Extra

450 L.E. is deposited at the beginning of every 20 days for 120 days at a simple interest rate of 15% annually. Find the interest.

Solution

Given: $A = 450,\quad n = \dfrac{120}{20} = 6,\quad r = 15\%$ *due, days*: $F = 120,\quad L = 20$

$$\sum A = (450 \times 6) + 450 \times 0.15 \times \frac{6}{2} \times \frac{120 + 20}{360} = 2700 + 78.75 = 2778.75$$

$$\text{Interest} = \sum A - (A \times n) = 2778.75 - 2700 = 78.75 \text{ L.E.}$$