Present Value and Discount

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Rules

Given		Keywords in the question
M_v	maturity value	*maturity value, face value*
t	time in years: days/360 or months/12	*before its maturity date by …*

Step	Rule	Keywords in the question
Discount	$M_v \times r_{\text{discount}} \times t$	*bank discount rate*
Commission	$M_v \times \dfrac{r_{\text{commission}}}{1000}$	*commission*
Collection fees	$M_v \times \dfrac{r_{\text{collection}}}{1000}$ never below the bank's minimum	*collection fees, at a minimum*
Agio	Discount + Commission + Collection	*Agio*
Present value	$P_v = M_v - \text{Agio}$	*present value, receives, gets*
Real discount rate	$r^* = \dfrac{\text{Agio}}{M_v \times t} \times 100$ always higher than r_{discount}, it includes all fees	*real discount rate*

Note. ‰ means per thousand: divide by 1000.

Q1

Azza discounted a note of maturity value \$20,000, 54 days before its maturity date, at a bank discount rate of 14%, collection fees 0.5 ‰ at a minimum of \$5 and commission 2 ‰. Compute:

a) Agio

b) Present value

c) Real discount rate

Solution

Given: $M_v = \$20{,}000$, $t = \frac{54}{360}$, $r_{\text{discount}} = 14\%$, $r_{\text{commission}} = 2\,‰$, $r_{\text{collection}} = 0.5\,‰$ *minimum \$5*

a) *Agio*

$$\text{Discount} = 20{,}000 \times 0.14 \times \frac{54}{360} = 420$$

$$\text{Commission} = 20{,}000 \times \frac{2}{1000} = 40$$

$$\text{Collection} = 20{,}000 \times \frac{0.5}{1000} = 10 \quad \textit{more than \$5, keep it}$$

$$\text{Agio} = 420 + 40 + 10 = 470$$

b) *Present value*

$$P_v = 20{,}000 - 470 = 19{,}530$$

c) *Real discount rate*

$$r^* = \frac{470}{20{,}000 \times \frac{54}{360}} \times 100 = \frac{470}{3{,}000} \times 100 = 15.67\%$$

Q2

A note of maturity value \$5,000 was discounted 2 months before its maturity date at a bank discount rate of 15%, collection fees 1 ‰ at a minimum of \$10 and commission 2 ‰. Compute:

a) Agio

b) Present value

c) Real discount rate

Solution

Given: $M_v = \$5{,}000,\ \ t = \frac{2}{12},\ \ r_{\text{discount}} = 15\%,\ \ r_{\text{commission}} = 2\,‰,\ \ r_{\text{collection}} = 1\,‰$ *minimum \$10*

a) *Agio*

$$\text{Discount} = 5{,}000 \times 0.15 \times \frac{2}{12} = 125$$

$$\text{Commission} = 5{,}000 \times \frac{2}{1000} = 10$$

$$\text{Collection} = 5{,}000 \times \frac{1}{1000} = 5 \ \rightarrow\ 10 \quad \textit{less than \$10, use the minimum}$$

$$\text{Agio} = 125 + 10 + 10 \ = \ \boxed{145}$$

b) *Present value*

$$P_v = 5{,}000 - 145 \ = \ \boxed{4{,}855}$$

c) *Real discount rate*

$$r^* = \frac{145}{5{,}000 \times \frac{2}{12}} \times 100 \ = \ \boxed{17.4\%}$$

Q3

A note was discounted 58 days before its maturity date at a bank discount rate of 13%, collection fees 1 ‰ and commission 2 ‰. Knowing that the maturity value is \$10,000, compute:

a) Agio

b) Present value

c) Real discount rate

Solution

Given: $M_v = \$10{,}000$, $t = \frac{58}{360}$, $r_{\text{discount}} = 13\%$, $r_{\text{commission}} = 2\,‰$, $r_{\text{collection}} = 1\,‰$ *no minimum*

a) *Agio*

$$\text{Discount} = 10{,}000 \times 0.13 \times \frac{58}{360} = 209.44$$

$$\text{Commission} = 10{,}000 \times \frac{2}{1000} = 20$$

$$\text{Collection} = 10{,}000 \times \frac{1}{1000} = 10$$

$$\text{Agio} = 209.44 + 20 + 10 = 239.44$$

b) *Present value*

$$P_v = 10{,}000 - 239.44 = 9{,}760.56$$

c) *Real discount rate*

$$r^* = \frac{239.44}{10{,}000 \times \frac{58}{360}} \times 100 = 14.86\%$$

Q4 Extra

On 12/4/2023 Omar discounted a note of maturity value \$36,000 due on 30/6/2023, at a bank discount rate of 12%, commission 2 ‰ and collection fees 1 ‰ at a minimum of \$50. Compute:

a) Agio

b) Present value

c) Real discount rate

Solution

Given: $M_v = \$36{,}000,\ r_{\text{discount}} = 12\%,\ r_{\text{commission}} = 2\,‰,\ r_{\text{collection}} = 1\,‰$ *minimum \$50*

First, count the days *skip the first day, count the last*

$$\text{Apr } 30 - 12 = 18,\ \text{May } 31,\ \text{Jun } 30$$

$$t = 18 + 31 + 30 = 79 \text{ days } = \frac{79}{360}$$

a) *Agio*

$$\text{Discount} = 36{,}000 \times 0.12 \times \frac{79}{360} = 948$$

$$\text{Commission} = 36{,}000 \times \frac{2}{1000} = 72$$

$$\text{Collection} = 36{,}000 \times \frac{1}{1000} = 36 \ \rightarrow\ 50$$ *less than \$50, use the minimum*

$$\text{Agio} = 948 + 72 + 50 = \boxed{1{,}070}$$

b) *Present value*

$$P_v = 36{,}000 - 1{,}070 = \boxed{34{,}930}$$

c) *Real discount rate*

$$r^* = \frac{1{,}070}{36{,}000 \times \frac{79}{360}} \times 100 = \frac{1{,}070}{7{,}900} \times 100 = \boxed{13.54\%}$$

Q5 Extra

Mona submitted two notes to a bank for discount at a bank discount rate of 15%, commission 3 ‰ and collection fees 1 ‰ at a minimum of \$4 per note:

1. A note of \$2,400 that matures in 3 months.
2. A note of \$6,000 that matures in 4 months.

Compute the Agio, the present value and the real discount rate of each note, then the total Agio and the total present value.

Solution

Given: $r_{\text{discount}} = 15\%$, $r_{\text{commission}} = 3\,‰$, $r_{\text{collection}} = 1\,‰$ *minimum \$4 per note*

	Note 1: \$2,400, 3 months	Note 2: \$6,000, 4 months
Discount	$2{,}400 \times 0.15 \times \frac{3}{12} = 90$	$6{,}000 \times 0.15 \times \frac{4}{12} = 300$
Commission	$2{,}400 \times \frac{3}{1000} = 7.2$	$6{,}000 \times \frac{3}{1000} = 18$
Collection	$2.4 \rightarrow 4$ *minimum*	6
Agio	101.2	324
P_v	2,298.8	5,676
r^*	$\frac{101.2}{2{,}400 \times \frac{3}{12}} \times 100 =$ 16.87%	$\frac{324}{6{,}000 \times \frac{4}{12}} \times 100 =$ 16.2%

$$\text{Total Agio} = 101.2 + 324 = 425.2$$

$$\text{Total } P_v = 8{,}400 - 425.2 = 7{,}974.8$$

Q6 Extra

Ali discounted a note of maturity value \$8,000, 90 days before its maturity date, and received \$7,740. Find the Agio and the real discount rate.

Solution

Given: $M_v = \$8{,}000,\quad P_v = \$7{,}740,\quad t = \dfrac{90}{360}$

$$\text{Agio} = M_v - P_v = 8{,}000 - 7{,}740 = 260$$

$$r^* = \frac{260}{8{,}000 \times \dfrac{90}{360}} \times 100 = \frac{260}{2{,}000} \times 100 = 13\%$$