Number Conversions and Operations

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Number systems

A number system is a way of writing numbers. Its **base** is how many digits it uses. We write the base small after the number, like $(101)_2$.

System	Base	Digits	Example
Binary	2	0 1	$(1011)_2$
Octal	8	0 1 2 3 4 5 6 7	$(57)_8$
Decimal	10	0 1 2 3 4 5 6 7 8 9	$(46)_{10}$
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F A = 10, B = 11, C = 12, D = 13, E = 14, F = 15	$(2E)_{16}$

Three parts

Part 1 — Q1 – Q6

From a smaller base to a bigger base

Binary → Decimal	2 → 10
Octal → Decimal	8 → 10
Binary → Octal	2 → 8
Binary → Hexadecimal	2 → 16

Special case

Hexadecimal → Decimal 16 → 10

bigger to smaller, but we still multiply

Part 2 — Q7 – Q12

From a bigger base to a smaller base

Decimal → Binary	10 → 2
Decimal → Octal	10 → 8
Octal → Binary	8 → 2
Hexadecimal → Binary	16 → 2

Special case

Decimal → Hexadecimal 10 → 16

smaller to bigger, but we still divide

Part 3 · Operations — Q13 – Q15

Binary addition and subtraction

For Dr. Ismail only in this quiz.

PART 1

From a smaller base to a bigger base

Q1 Binary → Decimal

Multiply each digit by the base raised to its position, then add.

Convert to decimal.

a) $(10110)_2$

b) $(1100101)_2$

c) $(101.11)_2$

d) $(11010.101)_2$

Solution

a) $(10110)_2 = (22)_{10}$

$= 1(2)^4 + 0(2)^3 + 1(2)^2 + 1(2)^1 + 0(2)^0 = 22$

b) $(1100101)_2 = (101)_{10}$

$= 1(2)^6 + 1(2)^5 + 0(2)^4 + 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 101$

c) $(101.11)_2 = (5.75)_{10}$

$= 1(2)^2 + 0(2)^1 + 1(2)^0 + 1(2)^{-1} + 1(2)^{-2} = 5.75$

d) $(11010.101)_2 = (26.625)_{10}$

$= 1(2)^4 + 1(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0$

$+ 1(2)^{-1} + 0(2)^{-2} + 1(2)^{-3} = 26.625$

Q2 Octal → Decimal

Multiply each digit by the base raised to its position, then add.

Convert to decimal.

a) $(47)_8$

b) $(365)_8$

c) $(12.4)_8$

d) $(207.14)_8$

Solution

a) $(47)_8 = (39)_{10}$

$= 4(8)^1 + 7(8)^0 = 39$

b) $(365)_8 = (245)_{10}$

$= 3(8)^2 + 6(8)^1 + 5(8)^0 = 245$

c) $(12.4)_8 = (10.5)_{10}$

$= 1(8)^1 + 2(8)^0 + 4(8)^{-1} = 10.5$

d) $(207.14)_8 = (135.1875)_{10}$

$= 2(8)^2 + 0(8)^1 + 7(8)^0 + 1(8)^{-1} + 4(8)^{-2} = 135.1875$

Q3 Hexadecimal → Decimal

Multiply each digit by the base raised to its position, then add.

Change letters to numbers first: A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Convert to decimal.

a) $(3B)_{16}$

b) $(1A7)_{16}$

c) $(F.8)_{16}$

d) $(A9.C)_{16}$

Solution

a) $(3B)_{16} = (59)_{10}$

$= 3(16)^1 + 11(16)^0 = 59$ *B = 11*

b) $(1A7)_{16} = (423)_{10}$

$= 1(16)^2 + 10(16)^1 + 7(16)^0 = 423$ *A = 10*

c) $(F.8)_{16} = (15.5)_{10}$

$= 15(16)^0 + 8(16)^{-1} = 15.5$ *F = 15*

d) $(A9.C)_{16} = (169.75)_{10}$

$= 10(16)^1 + 9(16)^0 + 12(16)^{-1} = 169.75$ *A = 10, C = 12*

Q4 Binary → Octal

Split into groups of 3 from the point. Read each group like Q1, with (2) and its powers.

Add zeros only at the two ends to complete a group (red zeros).

Convert to octal.

a) $(110101)_2$

b) $(1011100)_2$

c) $(101.11)_2$

d) $(11010.0101)_2$

Solution

a) $(110101)_2 = (65)_8$

$= 110 \mid 101$

$110 = 1(2)^2 + 1(2)^1 + 0(2)^0 = 6$

$101 = 1(2)^2 + 0(2)^1 + 1(2)^0 = 5$

b) $(1011100)_2 = (134)_8$

$= 001 \mid 011 \mid 100$

$001 = 0(2)^2 + 0(2)^1 + 1(2)^0 = 1$

$011 = 0(2)^2 + 1(2)^1 + 1(2)^0 = 3$

$100 = 1(2)^2 + 0(2)^1 + 0(2)^0 = 4$

c) $(101.11)_2 = (5.6)_8$

$= 101 \,.\, 110$

$101 = 1(2)^2 + 0(2)^1 + 1(2)^0 = 5$

$110 = 1(2)^2 + 1(2)^1 + 0(2)^0 = 6$

d) $(11010.0101)_2 = (32.24)_8$

$= 011 \mid 010 \,.\, 010 \mid 100$

$011 = 0(2)^2 + 1(2)^1 + 1(2)^0 = 3$

$010 = 0(2)^2 + 1(2)^1 + 0(2)^0 = 2$

$100 = 1(2)^2 + 0(2)^1 + 0(2)^0 = 4$

Q5 Binary → Hexadecimal

Split into groups of 4 from the point. Read each group like Q1, with (2) and its powers.

Add zeros only at the two ends to complete a group (red zeros).

Convert to hexadecimal.

a) $(11011110)_2$

b) $(1010111)_2$

c) $(1111.01)_2$

d) $(101101.1011)_2$

Solution

a) $(11011110)_2 = (DE)_{16}$

$= 1101 \mid 1110$

$1101 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 13 = D$

$1110 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 0(2)^0 = 14 = E$

b) $(1010111)_2 = (57)_{16}$

$= 0101 \mid 0111$

$0101 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 5$

$0111 = 0(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0 = 7$

c) $(1111.01)_2 = (F.4)_{16}$

$= 1111 . 0100$

$1111 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0 = 15 = F$

$0100 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 = 4$

d) $(101101.1011)_2 = (2D.B)_{16}$

$= 0010 \mid 1101 . 1011$

$0010 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 = 2$

$1101 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 13 = D$

$1011 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0 = 11 = B$

Q6 Check yourself

Convert $(11101.1)_2$ to:

a) decimal

b) octal

c) hexadecimal

d) Convert your answers to (b) and (c) back to decimal. Do you get (a)?

Solution

a) $(11101.1)_2 = (29.5)_{10}$

$$= 1(2)^4 + 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 + 1(2)^{-1} = 29.5$$

b) $(11101.1)_2 = (35.4)_8$

$= 011 \mid 101 \,.\, 100$

$011 = 0(2)^2 + 1(2)^1 + 1(2)^0 = 3$

$101 = 1(2)^2 + 0(2)^1 + 1(2)^0 = 5$

$100 = 1(2)^2 + 0(2)^1 + 0(2)^0 = 4$

c) $(11101.1)_2 = (1D.8)_{16}$

$= 0001 \mid 1101 \,.\, 1000$

$0001 = 0(2)^3 + 0(2)^2 + 0(2)^1 + 1(2)^0 = 1$

$1101 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 = 13 = \mathrm{D}$

$1000 = 1(2)^3 + 0(2)^2 + 0(2)^1 + 0(2)^0 = 8$

d) $(35.4)_8 = 3(8)^1 + 5(8)^0 + 4(8)^{-1} = 29.5$ *same as (a)*

$(1D.8)_{16} = 1(16)^1 + 13(16)^0 + 8(16)^{-1} = 29.5$ *same as (a)*

PART 2

From a bigger base to a smaller base

Q7 Decimal → Binary

1 Whole part: divide by 2, read **up** 2 Fraction: multiply by 2, read **down**

Solution

a) $(13)_{10} = (1101)_2$

2	13	6	.5 × 2 = **1**
2	6	3	.0 × 2 = **0**
2	3	1	.5 × 2 = **1**
2	1	0	.5 × 2 = **1**
	0	stop: we reached 0	

↑ up

b) $(46)_{10} = (101110)_2$

2	46	23	.0 × 2 = **0**
2	23	11	.5 × 2 = **1**
2	11	5	.5 × 2 = **1**
2	5	2	.5 × 2 = **1**
2	2	1	.0 × 2 = **0**
2	1	0	.5 × 2 = **1**
	0	stop: we reached 0	

↑ up

c) $(10.5)_{10} = (1010.1)_2$

2	10	5	.0 × 2 = **0**
2	5	2	.5 × 2 = **1**
2	2	1	.0 × 2 = **0**
2	1	0	.5 × 2 = **1**
	0	stop: we reached 0	

↑ up

0.5 × 2 = **1** ↓ down

stop: no fraction left

d) $(19.375)_{10} = (10011.011)_2$

2	19	9	.5 × 2 = **1**
2	9	4	.5 × 2 = **1**
2	4	2	.0 × 2 = **0**
2	2	1	.0 × 2 = **0**
2	1	0	.5 × 2 = **1**
	0	stop: we reached 0	

↑ up

0.375 × 2 = **0**.75
0.75 × 2 = **1**.5
0.5 × 2 = **1**

↓ down

stop: no fraction left

Q8 Decimal → Octal

1. Whole part: divide by 8, read **up**
2. Fraction: multiply by 8, read **down**

Solution

a) $(59)_{10} = (73)_8$

8	59	7	$.375 \times 8 = \mathbf{3}$ ↑ up
8	7	0	$.875 \times 8 = \mathbf{7}$
	0		stop: we reached 0

b) $(250)_{10} = (372)_8$

8	250	31	$.25 \times 8 = \mathbf{2}$ ↑ up
8	31	3	$.875 \times 8 = \mathbf{7}$
8	3	0	$.375 \times 8 = \mathbf{3}$
	0		stop: we reached 0

c) $(9.25)_{10} = (11.2)_8$

8	9	1	$.125 \times 8 = \mathbf{1}$ ↑ up
8	1	0	$.125 \times 8 = \mathbf{1}$
	0		stop: we reached 0

$0.25 \times 8 = \mathbf{2}$ ↓ down
stop: no fraction left

d) $(87.125)_{10} = (127.1)_8$

8	87	10	$.875 \times 8 = \mathbf{7}$ ↑ up
8	10	1	$.25 \times 8 = \mathbf{2}$
8	1	0	$.125 \times 8 = \mathbf{1}$
	0		stop: we reached 0

$0.125 \times 8 = \mathbf{1}$ ↓ down
stop: no fraction left

Q9 Decimal → Hexadecimal

1 Whole part: divide by 16, read **up** 2 Fraction: multiply by 16, read **down**

Remainders 10 to 15 become letters: A = 10, B = 11, C = 12, D = 13, E = 14, F = 15

Solution

a) $(47)_{10} = (2F)_{16}$

16 | 47	2	.9375 × 16 = 15 = **F**	↑ up
16 | 2	0	.125 × 16 = **2**	
0	stop: we reached 0		

b) $(300)_{10} = (12C)_{16}$

16 | 300	18	.75 × 16 = 12 = **C**	↑ up
16 | 18	1	.125 × 16 = **2**	
16 | 1	0	.0625 × 16 = **1**	
0	stop: we reached 0		

c) $(26.5)_{10} = (1A.8)_{16}$

16 | 26	1	.625 × 16 = 10 = **A**	↑ up
16 | 1	0	.0625 × 16 = **1**	
0	stop: we reached 0		

0.5 × 16 = **8** ↓ down

stop: no fraction left

d) $(255.25)_{10} = (FF.4)_{16}$

16 | 255	15	.9375 × 16 = 15 = **F**	↑ up
16 | 15	0	.9375 × 16 = 15 = **F**	
0	stop: we reached 0		

0.25 × 16 = **4** ↓ down

stop: no fraction left

Q10 Octal → Binary

1. Write each digit as 3 bits, with (2) and its powers
2. Keep the point in its place

Red zeros complete 3 bits. In the answer, drop zeros at the two ends.

Solution

a) $(63)_8 = (110011)_2$

$6 = 1(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 110$

$3 = 0(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 011$

$= (110 \mid 011)_2$

b) $(470)_8 = (100111000)_2$

$4 = 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 100$

$7 = 1(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 111$

$0 = 0(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 000$

$= (100 \mid 111 \mid 000)_2$

c) $(7.2)_8 = (111.01)_2$

$7 = 1(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 111$

$2 = 0(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 010$

$= (111\,.\,010)_2$ *drop end zeros*

d) $(31.64)_8 = (11001.1101)_2$

$3 = 0(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 011$

$1 = 0(2)^2 + 0(2)^1 + 1(2)^0 \rightarrow 001$

$6 = 1(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 110$

$4 = 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 100$

$= (011 \mid 001\,.\,110 \mid 100)_2$ *drop end zeros*

Q11 Hexadecimal → Binary

1. Write each digit as 4 bits, with (2) and its powers
2. Keep the point in its place

Red zeros complete 4 bits. In the answer, drop zeros at the two ends.

Solution

a) $(9F)_{16} = (10011111)_2$

$$9 = 1(2)^3 + 0(2)^2 + 0(2)^1 + 1(2)^0 \rightarrow 1001$$
$$F = 15 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 1111$$
$$= (1001 \mid 1111)_2$$

b) $(2B4)_{16} = (1010110100)_2$

$$2 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 0010$$
$$B = 11 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 1(2)^0 \rightarrow 1011$$
$$4 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 0100$$
$$= (0010 \mid 1011 \mid 0100)_2 \quad \textit{drop end zeros}$$

c) $(C.4)_{16} = (1100.01)_2$

$$C = 12 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 1100$$
$$4 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 0100$$
$$= (1100\,.\,0100)_2 \quad \textit{drop end zeros}$$

d) $(5E.A)_{16} = (1011110.101)_2$

$$5 = 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 \rightarrow 0101$$
$$E = 14 = 1(2)^3 + 1(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 1110$$
$$A = 10 = 1(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 1010$$
$$= (0101 \mid 1110\,.\,1010)_2 \quad \textit{drop end zeros}$$

Q12 Check yourself

Convert $(45.75)_{10}$ to (a) binary, (b) octal and (c) hexadecimal. Then (d) convert your answers to (b) and (c) to binary, the way you did in Q10 and Q11. Do you get (a)?

Solution

a) $(45.75)_{10} = (101101.11)_2$

2 | 45	22	.5 × 2 = **1**	↑ up
2 | 22	11	.0 × 2 = **0**	
2 | 11	5	.5 × 2 = **1**	
2 | 5	2	.5 × 2 = **1**	
2 | 2	1	.0 × 2 = **0**	
2 | 1	0	.5 × 2 = **1**	
0	stop: we reached 0		

0.75 × 2 = **1**.5
0.5 × 2 = **1** ↓ down
stop: no fraction left

b) $(45.75)_{10} = (55.6)_8$

8 | 45	5	.625 × 8 = **5**	↑ up
8 | 5	0	.625 × 8 = **5**	
0	stop: we reached 0		

0.75 × 8 = **6** ↓ down
stop: no fraction left

c) $(45.75)_{10} = (2D.C)_{16}$

16 | 45	2	.8125 × 16 = 13 = **D**	↑ up
16 | 2	0	.125 × 16 = **2**	
0	stop: we reached 0		

0.75 × 16 = **12** = **C** ↓ down
stop: no fraction left

d) same as (a)

$(55.6)_8 = (101101.11)_2$

$5 = 1(2)^2 + 0(2)^1 + 1(2)^0 \rightarrow 101$

$6 = 1(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 110$

$= (101 \mid 101.110)_2$ *drop end zeros*

$(2D.C)_{16} = (101101.11)_2$

$2 = 0(2)^3 + 0(2)^2 + 1(2)^1 + 0(2)^0 \rightarrow 0010$

$D = 13 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0 \rightarrow 1101$

$C = 12 = 1(2)^3 + 1(2)^2 + 0(2)^1 + 0(2)^0 \rightarrow 1100$

$= (0010 \mid 1101.1100)_2$ *drop end zeros*

PART 3

Operations: binary addition and subtraction

For Dr. Ismail only in this quiz.

Q13 Binary addition

Write the numbers under each other, lined up on the right. Add one column at a time, from right to left.

$0 + 0 = \mathbf{0}$

$0 + 1 = \mathbf{1}$

$1 + 1 = \mathbf{0}$ carry 1 to the next column

$1 + 1 + 1 = \mathbf{1}$ carry 1 to the next column

Add.

a) $(100011)_2 + (100110)_2$

b) $(111110)_2 + (10111)_2$

c) $(1110111)_2 + (110110)_2$

d) $(11111)_2 + (11011)_2$

Solution

a) $(100011)_2 + (100110)_2$

$= (1001001)_2$

carry	1			1	1		
		1	0	0	0	1	1
		1	0	0	1	1	0 +
	1	**0**	**0**	**1**	**0**	**0**	**1**

check in decimal: 35 + 38 = 73

b) $(111110)_2 + (10111)_2$

$= (1010101)_2$

carry	1	1	1	1	1		
		1	1	1	1	1	0
			1	0	1	1	1 +
	1	**0**	**1**	**0**	**1**	**0**	**1**

check in decimal: 62 + 23 = 85

c) $(1110111)_2 + (110110)_2$

$= (10101101)_2$

carry	1	1	1		1	1		
		1	1	1	0	1	1	1
			1	1	0	1	1	0 +
	1	**0**	**1**	**0**	**1**	**1**	**0**	**1**

check in decimal: 119 + 54 = 173

d) $(11111)_2 + (11011)_2$

$= (111010)_2$

carry	1	1	1	1	1		
		1	1	1	1	1	
		1	1	0	1	1	+
	1	**1**	**1**	**0**	**1**	**0**	

check in decimal: 31 + 27 = 58

Q14 Binary subtraction

Write the bigger number on top, lined up on the right. Subtract one column at a time, from right to left.

$0 - 0 = \mathbf{0}$

$1 - 0 = \mathbf{1}$

$1 - 1 = \mathbf{0}$

$0 - 1 = \mathbf{1}$ borrow 1 from the next column on the left

Subtract.

a) $(10101)_2 - (1010)_2$

b) $(101001)_2 - (11011)_2$

c) $(100001)_2 - (11)_2$

d) $(1010000)_2 - (101011)_2$

Solution

a) $(10101)_2 - (1010)_2 = (1011)_2$

borrow	1		1			
	1	0	1	0	1	
		1	0	1	0	−
	0	**1**	**0**	**1**	**1**	

check in decimal: 21 − 10 = 11

b) $(101001)_2 - (11011)_2 = (1110)_2$

borrow	1	1	1	1			
	1	0	1	0	0	1	
		1	1	0	1	1	−
	0	**0**	**1**	**1**	**1**	**0**	

check in decimal: 41 − 27 = 14

c) $(100001)_2 - (11)_2 = (11110)_2$

borrow	1	1	1	1		
	1	0	0	0	0	1
					1	1 −
	0	**1**	**1**	**1**	**1**	**0**

check in decimal: 33 − 3 = 30

d) $(1010000)_2 - (101011)_2 = (100101)_2$

borrow	1		1	1	1	1	
	1	0	1	0	0	0	0
		1	0	1	0	1	1 −
	0	**1**	**0**	**0**	**1**	**0**	**1**

check in decimal: 80 − 43 = 37

Q15 Check yourself

Let $A = (10110)_2$ and $B = (1011)_2$.

a) Find $A + B$.

b) Subtract B from your answer to (a). Do you get A back?

c) Convert A, B and your answer to (a) to decimal, like Q1. Does the sum still match?

Solution

a) $(10110)_2 + (1011)_2 = (100001)_2$

carry	1	1	1	1		
		1	0	1	1	0
			1	0	1	1 +
	1	**0**	**0**	**0**	**0**	**1**

check in decimal: 22 + 11 = 33

b) $(100001)_2 - (1011)_2 = (10110)_2$

borrow	1	1	1	1		
	1	0	0	0	0	1
			1	0	1	1 −
	0	**1**	**0**	**1**	**1**	**0**

check in decimal: 33 − 11 = 22

c) $A = 22,\ B = 11,\ A + B = (100001)_2 = 33$ *and $22 + 11 = 33$, so it matches*